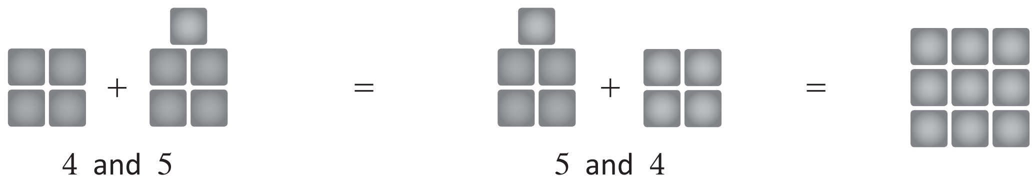
Arithmetic laws

There are three laws of arithmetic that can be powerful tools to help with calculations.

1. Commutative laws

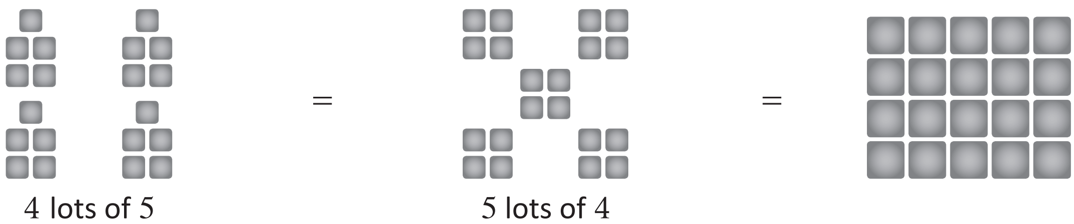
The order that we add or multiply numbers can be switched without changing the answer.

- Addition $a + b = b + a$



$$4 + 5 = 5 + 4 = 9$$

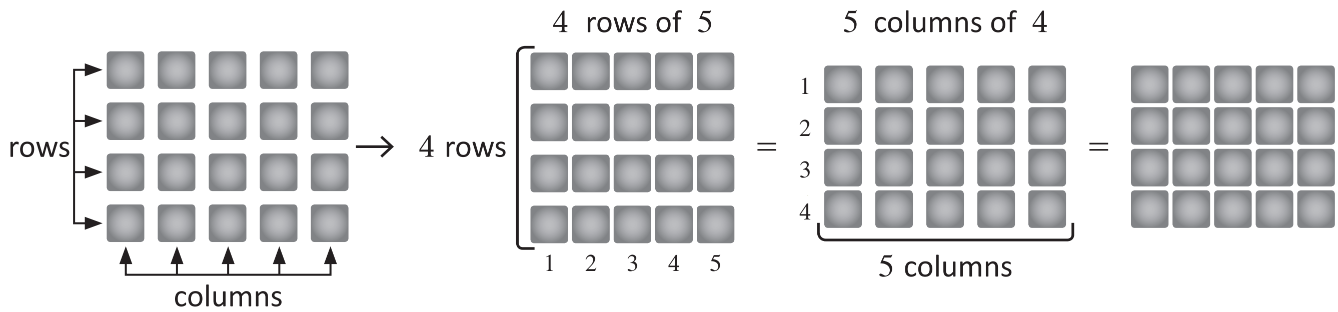
- Multiplication $a \times b = b \times a$



$$4 \times 5 = 5 \times 4 = 20$$

The commutative law for multiplication can be shown also in terms of rows and columns.

$$rows \times columns = columns \times rows$$



$$4 \times 5 = 5 \times 4 = 20$$

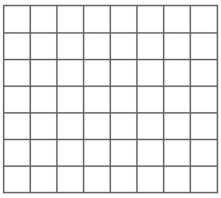
The commutative law does not work for subtraction or division as the order of the terms is important.

- $4 - 5 \neq 5 - 4 \rightarrow 4 - 5 = -1$ and $5 - 4 = 1$
- $4 \div 5 \neq 5 \div 4 \rightarrow 4 \div 5 = 0.8$ and $5 \div 4 = 1.25$

Commutative laws

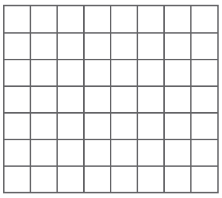
1. Shade groups of boxes to match these descriptions and write the calculation they represent.

a. Two rows of four



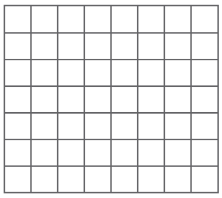
= ___ × ___

b. Four columns of two



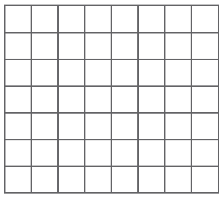
= ___ × ___

c. Four rows of two



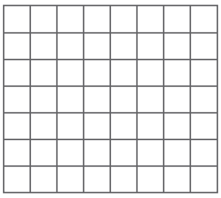
= ___ × ___

d. Two columns of four



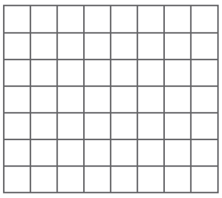
= ___ × ___

e. Four columns of one



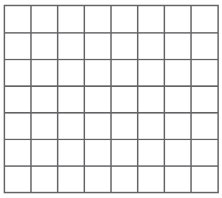
= ___ × ___

f. Three rows of six



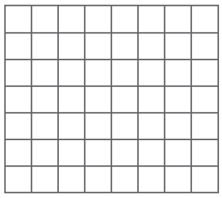
= ___ × ___

g. Two rows of seven



= ___ × ___

h. Four rows of one



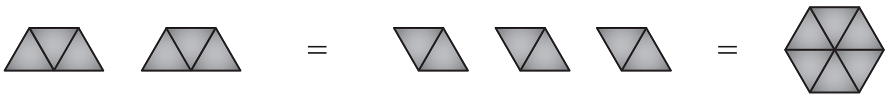
= ___ × ___

2. Fill in the mathematical sentences for each of these diagrams showing the commutative law.

a.

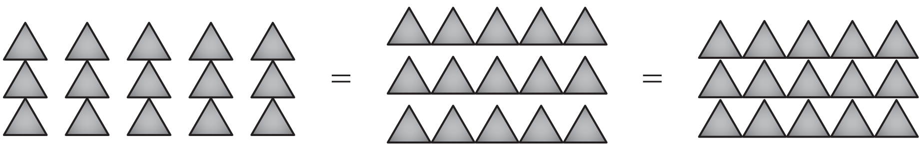
___ + ___ = ___ + ___ = ___

b.



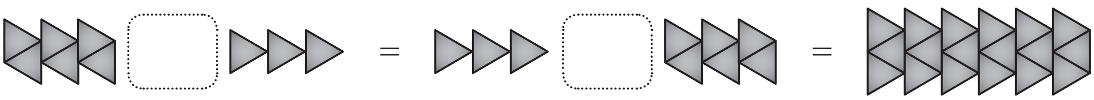
___ × ___ = ___ × ___ = ___

c.



___ × ___ = ___ × ___ = ___

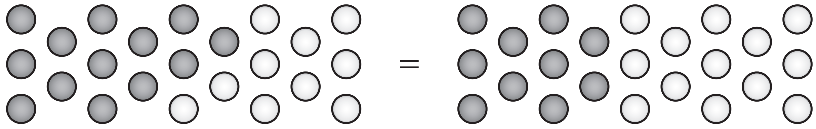
d.



___ = ___ = ___

Commutative laws

3. Write the expression for the commutative law represented by this diagram.



=

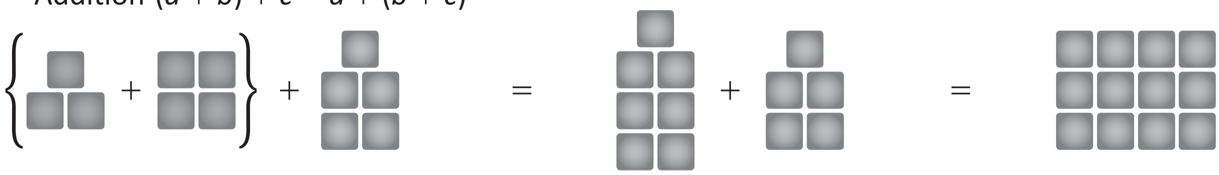
4. Draw two different diagrams below to demonstrate that $2 \times 6 = 6 \times 2 = 12$.

5. Earn yourself an awesome stamp for this one.
Draw all four different pairs of diagrams that represent the commutative law for multiplication with an answer of 24.

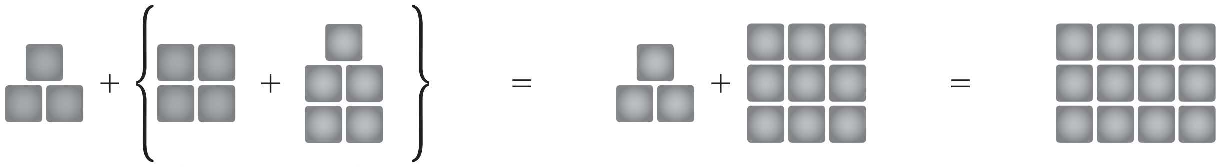
2. Associative laws

The numbers we group together when adding or multiplying can change without changing the answer.

- Addition $(a + b) + c = a + (b + c)$

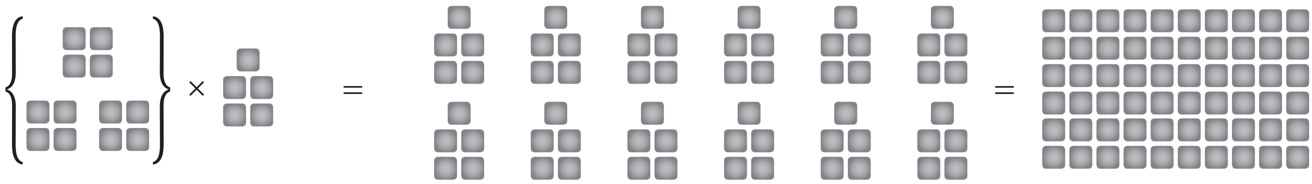


$(3 + 4) + 5 = 7 + 5 = 12$



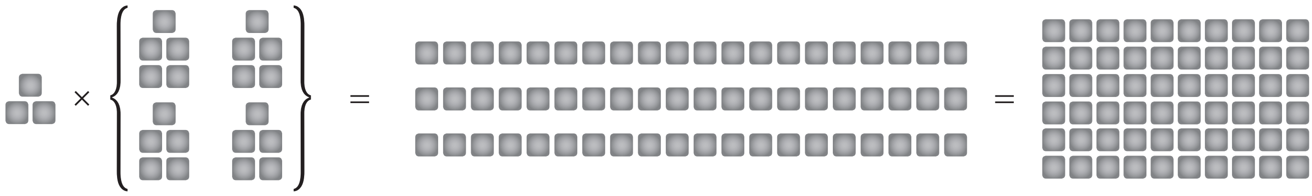
$3 + (4 + 5) = 3 + 9 = 12$

- Multiplication $(a \times b) \times c = a \times (b \times c)$



3 lots of 4 × 5 = 12 lots of 5

$(3 \times 4) \times 5 = 12 \times 5 = 60$



3 × 4 lots of 5 = 3 lots of 20

$3 \times (4 \times 5) = 3 \times 20 = 60$

The associative law can make adding/multiplying terms easier.

$$\begin{aligned} 31 + 25 + 9 &= (31 + 9) + 25 \\ &= 40 + 25 \\ &= 65 \end{aligned}$$

$$\begin{aligned} 13 \times 25 \times 4 &= 13 \times (25 \times 4) \\ &= 13 \times 100 \\ &= 1300 \end{aligned}$$

What about subtraction or division?
There are only a few very special cases where the associative law works for subtraction and division.

When a 0 is involved like this:

- $(6 - 2) - 0 = 6 - (2 - 0)$
- $(0 \div 7) \div 5 = 0 \div (7 \div 5)$

When a 1 is involved like this:

- $(18 \div 3) \div 1 = 18 \div (3 \div 1)$

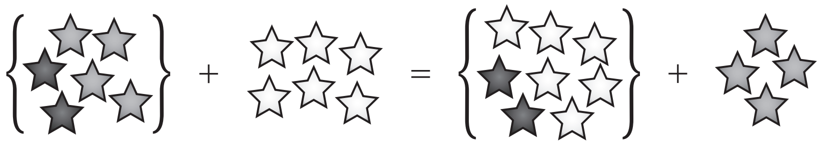
In all other cases, the associative law does not work for subtraction and division.

- $(3 - 4) - 5 \neq 3 - (4 - 5) \rightarrow (3 - 4) - 5 = -6$ and $3 - (4 - 5) = 4$
- $(3 \div 4) \div 5 \neq 3 \div (4 \div 5) \rightarrow (3 \div 4) \div 5 = 0.15$ and $3 \div (4 \div 5) = 3.75$

Associative laws

1 Complete these equations for the associative law of addition shown in each diagram.

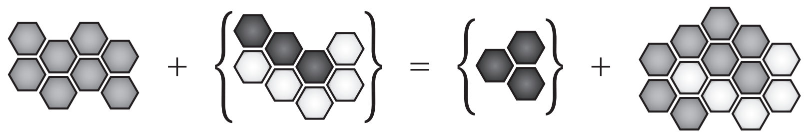
a

 = 12

$$\{\boxed{2} + \boxed{4}\} + \boxed{6} = \{\boxed{2} + \boxed{6}\} + \boxed{4}$$

$$\boxed{6} + \boxed{6} = \boxed{8} + \boxed{4} = \boxed{12}$$

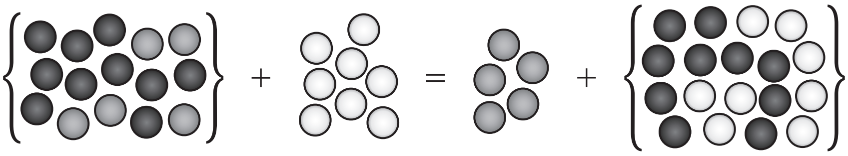
b



$$\square + \{\square + \square\} = \square + \{\square + \square\}$$

$$\square + \square = \square + \square = \square$$

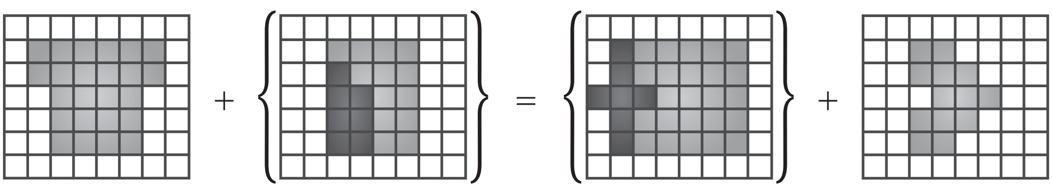
c



$$\{\square + \square\} + \square = \square + \{\square + \square\}$$

$$\square + \square = \square + \square = \square$$

d



$$\square + \{\square + \square\} = \{\square + \square\} + \square$$

$$\square + \square = \square + \square = \square$$

Associative laws

2 The associative law lets us choose what we want to do first to make calculations easier and quicker.

For example:

$14 + 9 + 16$ is made easier by adding the 14 and 16 together first to make it $9 + 30 = 39$

Pairing up terms that make nicer whole numbers before adding to other terms is a great trick.

Use the associative law for addition to simplify these calculations:

a. $25 + 91 + 75 = \{\square + \square\} + 91$

$= \square + 91$

$= \square$

b. $83 + 52 + 18 = \{\square + \square\} + 83$

$= \square + 83$

$= \square$

c. $122 + 163 + 37 = \{\square + \square\} + \square$

$= \square + \square$

$= \square$

d. $102 + 43 + 25 = \{\square + \square\} + \square$

$= \square + \square$

$= \square$

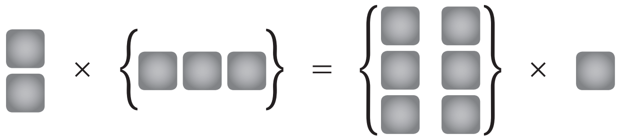
e. $37 + 14 + 56 + 23 =$

f. $111 + 80 + 19 + 45 =$

Associative laws

3 Complete these equations for the associative law of multiplication shown in each diagram.

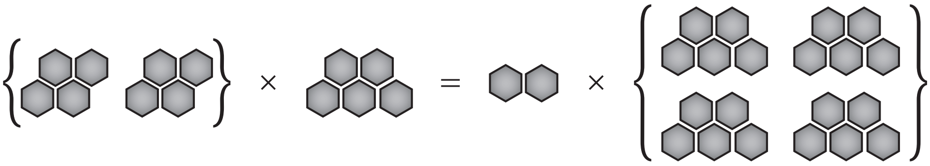
a

 = 6

$$\square \times \{\square \times \square\} = \{\square \times \square\} \times \square$$

$$\square \times \square = \square \times \square = 6$$

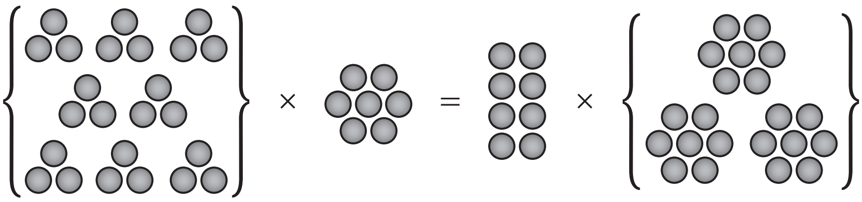
b

 = 40

$$\{\square \times \square\} \times \square = \square \times \{\square \times \square\}$$

$$\square \times \square = \square \times \square = 40$$

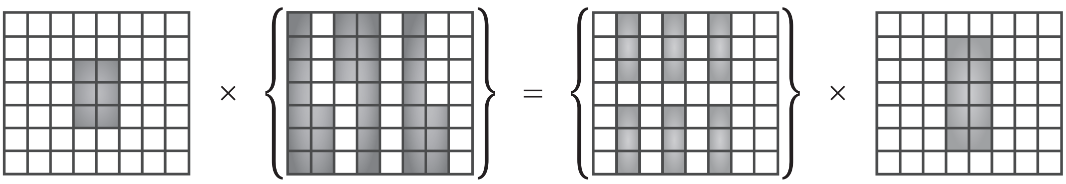
c

 = 168

$$\{\square \times \square\} \times \square = \square \times \{\square \times \square\}$$

$$\square \times \square = \square \times \square = 168$$

d



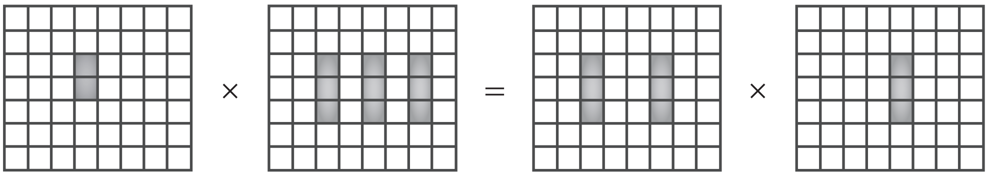
$$\square \times \{\square \times \square\} = \{\square \times \square\} \times \square$$

$$\square \times \square = \square \times \square = \square$$

Associative laws

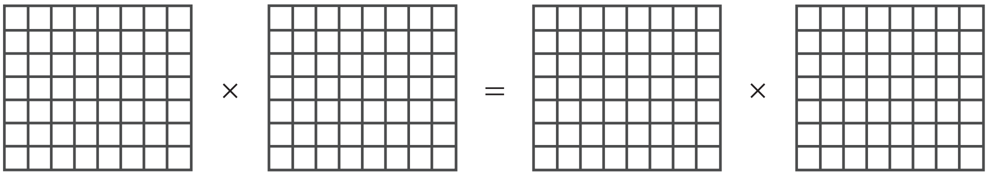
4 Shade groups of squares in these grids to match the associative law of multiplication underneath.

a



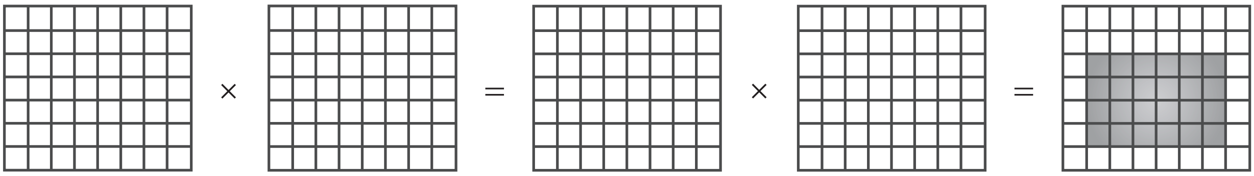
$2 \times (3 \times 3) = (2 \times 3) \times 3$

b



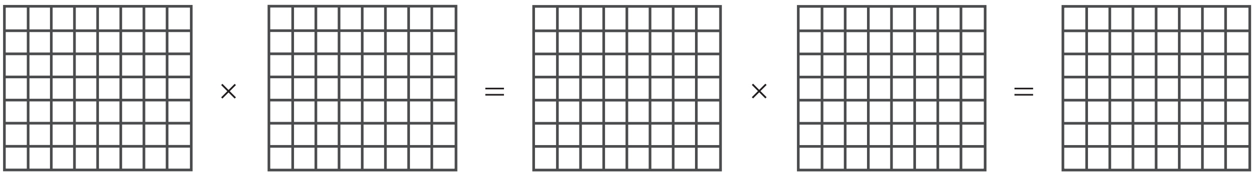
$4 \times (3 \times 6) = (4 \times 3) \times 6$

c



$4 \times (2 \times 3) = (4 \times 2) \times 3 = 24$

d



$3 \times (5 \times 2) = (3 \times 5) \times 2 = 30$

5 Use the associative law for multiplication to simplify these calculations:

a $5 \times 28 \times 20 = \{\square \times \square\} \times 28$

$= \square \times 28$

$= \square$

b $12 \times 50 \times 7 = \{\square \times \square\} \times 7$

$= \square \times 7$

$= \square$

c $4 \times 9 \times 75 = \{\square \times \square\} \times \square$

$= \square \times \square$

$= \square$

d $15 \times 12 \times 11 = \{\square \times \square\} \times \square$

$= \square \times \square$

$= \square$

Associative laws

6 Tick the box 'true' or 'false' to show which of these subtraction or division statements are special cases where the associative law does work.

a $(12 \div 4) \div 1 = 12 \div (4 \div 1)$ ☐ True ☐ False
b $(5 \div 0) \div 3 = 5 \div (0 \div 3)$ ☐ True ☐ False
c $(0 - 4) - 3 = 0 - (4 - 3)$ ☐ True ☐ False
d $(0 \div 16) \div 2 = 0 \div (16 \div 2)$ ☐ True ☐ False
e $(20 - 11) - 0 = 20 - (11 - 0)$ ☐ True ☐ False
f $(5 \div 1) \div 5 = 5 \div (1 \div 5)$ ☐ True ☐ False
g $(0 \div 1) \div 1 = 0 \div (1 \div 1)$ ☐ True ☐ False
h $(8 - 0) - 1 = 8 - (0 - 1)$ ☐ True ☐ False
i $(1 - 1) - 1 = 1 - (1 - 1)$ ☐ True ☐ False
j $(0 - 1) - 0 = 0 - (1 - 0)$ ☐ True ☐ False

7 Larger strings of numbers can also have the associative law applied to them.

Group terms together in these expressions that make the calculation much easier.

a $12 + 34 + 8 + 4 + 2$

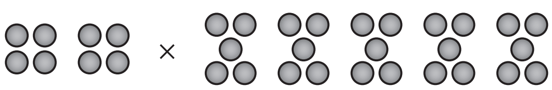
Explain why you grouped the numbers you did.

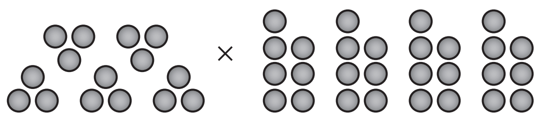
b $12 \times 2 \times 5 \times 3$

c Use the associative law to rearrange into three pairs to make the calculation easier.

$$23 + 11 + 37 + 24 + 16 + 9$$

8 Write an expression that represents these dot diagrams and use the associative law to simplify.

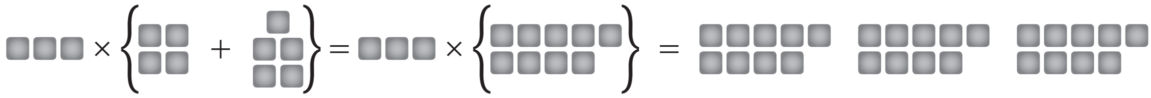
a 

b 

3. Distributive law

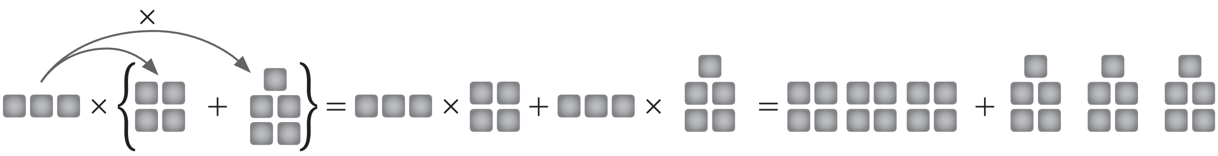
This law allows you to spread a multiplication out (or expand) into smaller parts to make it simpler.

- Using the order of operations and calculating the brackets first:



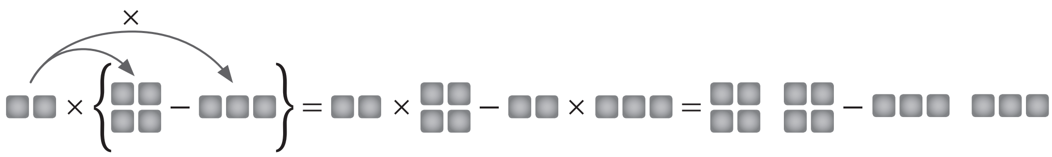
$$3 \times (4 + 5) = 3 \times (9) = 27$$

- Distributing the multiplication to each term within the brackets:



$$3 \times (4 + 5) = 3 \times 4 + 3 \times 5 = 12 + 15 = 27$$

Also works if a subtraction is inside the brackets.



$$2 \times (4 - 3) = 2 \times 4 - 2 \times 3 = 8 - 6 = 2$$

So the distributive law is:

$$a \times (b + c) = a \times b + a \times c$$

These signs must always match

$$a \times (b - c) = a \times b - a \times c$$

These signs must always match

Using this law in reverse makes the multiplication of large numbers easy to calculate mentally.

For example, we can use the distributive law to calculate 127×4 using either of these:

$120 \times 4 + 7 \times 4$ or $130 \times 4 - 3 \times 4$

$$\begin{aligned} 127 \times 4 &= (120 + 7) \times 4 \\ &= 120 \times 4 + 7 \times 4 \\ &= 480 + 28 \\ &= 508 \end{aligned}$$

$$\begin{aligned} 127 \times 4 &= (130 - 3) \times 4 \\ &= 130 \times 4 - 3 \times 4 \\ &= 520 - 12 \\ &= 508 \end{aligned}$$

You could have split 127 up into any sum/subtraction you found easiest to use.

Distributive law

1 Fill in the missing values for each of these examples of the distributive law.

a $3 \times \left\{5 + \boxed{}\right\} = 3 \times \boxed{} + 3 \times 6$

b $\boxed{} \times \left\{8 - 4\right\} = 7 \times 8 - 7 \times \boxed{}$

c $6 \times 13 + 6 \times 9 = \boxed{} \times \left\{\boxed{} + \boxed{}\right\}$

d $\boxed{} \times 7 - \boxed{} \times 10 = 15 \times \left\{7 - \boxed{}\right\}$

e $\boxed{} \times \left\{9 + \boxed{}\right\} = 45 + 5 \times 11$

f $12 \times \left\{\boxed{} - \boxed{}\right\} = 36 - 12$

2 **a** Use the distributive law to expand these multiplications:

α) $5 \times \left\{8 + 2\right\} = 5 \times \boxed{} + 5 \times \boxed{}$

$= \boxed{} + \boxed{}$

$= \boxed{}$

β) $5 \times \left\{4 + 6\right\} = 5 \times \boxed{} + 5 \times \boxed{}$

$= \boxed{} + \boxed{}$

$= \boxed{}$

b Explain why both have the same value even though the terms in the brackets are different.

c Write another similar expression to those in **a** that will also give the same answer.

3 Use the distributive law to simplify and evaluate these multiplications:

a $6 \times 25 = 6 \times \left\{\boxed{} + \boxed{}\right\}$

$= 6 \times \boxed{} + 6 \times \boxed{}$

$= \boxed{} + \boxed{}$

$= \boxed{}$

b $8 \times 98 = 8 \times \left\{\boxed{} - \boxed{}\right\}$

$= 8 \times \boxed{} - 8 \times \boxed{}$

$= \boxed{} - \boxed{}$

$= \boxed{}$

c $11 \times 32 = 11 \times \left\{\boxed{} + \boxed{}\right\}$

$= 11 \times \boxed{} + 11 \times \boxed{}$

$= \boxed{} + \boxed{}$

$= \boxed{}$

d $15 \times 19 = 15 \times \left\{\boxed{} - \boxed{}\right\}$

$= 15 \times \boxed{} - 15 \times \boxed{}$

$= \boxed{} - \boxed{}$

$= \boxed{}$

Distributive law

4 You can apply the distributive law more than once to simplify a calculation:

For example:

$$
\begin{aligned}
25 \times 59 &= 25 \times (60 - 1) \\
&= 25 \times 60 - 25 \times 1 \\
&= 25 \times 60 - 25
\end{aligned}
\qquad \text{OR} \qquad
= 25 \times 60 - 25
$$

$$
\begin{aligned}
&= (20 + 5) \times 60 - 25 \\
&= 20 \times 60 + 5 \times 60 - 25 \\
&= 1200 + 300 - 25 \\
&= 1475
\end{aligned}
\qquad\qquad
\begin{aligned}
&= 25 \times (20 + 40) - 25 \\
&= 25 \times 20 + 25 \times 40 - 25 \\
&= 500 + 1000 - 25 \\
&= 1475
\end{aligned}
$$

Apply the distributive law twice to simplify and calculate these:

a 14×37

b 45×82

c 22×75

d 25×112

e 83×35

f 120×108